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Harvesting Energy Density Performance Of Cantilevered Piezoelectric Transducers

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Abstract: To estimate theharvesting performance of piezoelectrictransducers under finite volume, the models of unimorph cantilevers are established and then the analyses of influences of external and internal factors on harvesting energy density is proposed. The results show that the more heavy the tip mass is, the better output performance it would turn out at the first natural frequency, which is contrary to output capacity at the second natural frequency. Moreover, the harvesting voltage density is proportional to the acceleration of the base excitation. It is found that output voltage increases along with the growth of load resistance and there exists an optimal load resistance which leads to the most desirable output power density.Besides external factors, internal factors such as thickness ratio and material properties of substrate can be of great significance to output performance. The findings exhibit that an optimum thickness ratio corresponding to the most desirable output voltage density would emerge either aluminum or copper substrate involved in energy harvesting systems and further studies apparently show that copper substrate generates higher performance in comparison with aluminum substrate.

Keywords: piezoelectric transducers, unimorph cantilever, electric energy density, external and internal factors

I. Introduction

Harvesting ambient vibrational energy to power the wireless sensors devices had been proved to be feasible in the past few decades. The energy conversionmechanisms that can be used to convert ambient vibrationsinto electricity energy are electromagnetic [1-3], electrostatic [4, 5], and piezoelectric[6-8] conversion mechanisms, respectively.Due to higher electromechanical coupling performance, greater voltage output, needing no external power supply and being compatible with MEMS technology, the self-powered technology based on piezoelectric transduction has attracted a great deal of attentionsas exhibited in some published literatures [9-12] that directly focusing on piezoelectric energy harvesting.

Many researchers adoptthecantilevered piezoelectric energy transducers togenerateelectrical power frompiezoelectricmaterials. The reason is that thecantilever is of simple structures and can be manufactured and fabricatedre latively easily.In terms of this type of energy transducers,one or two layer so fpiezoelectric material areusually attached to ametalsubstratelayer,and we named the munimorphorbimorph cantilevers.Energy transducers normally operate in the 31mode,in which the bending stress developed in the long it udinaldirection(1-direction),produces apotential difference across the thickness direction(3-direction)of the piezoelectriclayer. Normally,the natural frequency of anenergy transducer has to be tuned around the external in centives frequencytooperateatresonance state andmaximizethe energy output.Consequently, a tip mass is normally attached to the free end of the cantilevered piezoelectric energy transducers for the purpose of reducing thenaturalfrequency of energy harvesting systems.

Researchers have obtained highly significant achievements in recent years, yet certain aspects haven't received particular attentions so far.As to the generation performance of piezoelectric harvesters, it was mainly characterized via index such as output voltage, yet few evaluation methods based on voltage densityperformance had been proposed.However, the miniaturization of the self-powered system is also of great importance simultaneously [10, 13]. Kang L H [14] made investigation on performance of piezoelectric unimorph with mechanically prestressed substrate (PUMPS) evaluated via actuation displacement and force, and the findings suggest that PUMPS is of better performance than conventional ones. However, due to large dimension and low integration level caused by extra curved space under piezoelectric unimorph, PUMPS is not desirablefor MEMS technology. In other words, it is anextremely important method for bettercompatibility with MEMS technology to obtain maximum electric energy in smallest volume, and generation performance in unit volume of piezoelectric energy harvester which are considered as harvesting electric density parameters. In this paper, the harvesting electric density performance is brought forward as energy output assessmentindex on the basis of analysis on external and internal factors. Meanwhile, this paper providescertain theoretical references for energy harvesting technology using piezoelectric materialswith cantilever configuration.

II. Modeling Of Coupled Piezoelectric Transducers

In this article, unimorph forms of transducers are taken into account, as shown in figure 1. The unimorph consists of two layers: PZT and substrate layer, a tip mass is attached to the free end of the beam and base excitation is applied to the fix end of the piezoelectric cantilever. Meanwhile, an external load resistanceR_{load} =12 kΩ is presented to form an electric circuit.

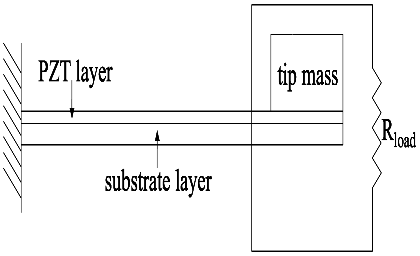


Figure 1. Unimorph piezoelectric energy transducer with a tip mass.

Fora uniformbeamthatundergoesundampedfreevibration,thegoverningequationofmotioncanbe expressed as

$$\frac{\partial^2 M(x,t)}{\partial x^2}+m\frac{\partial^2 w(x,t)}{\partial t^2}=f_0(x,t) \tag{1}$$

Where $M(x,t)$ is the internalbending moment, m is the mass per unit length of thebeam and $f_0(x,t)$ is the external forceper unit length that applied to the beam.

The displacement of the beam can be obtained as [15]

$$w(x,t)=w_b(x,t)+w_{rel}(x,t) \tag{2}$$

Where $w_b(x,t)$ and $w_{rel}(x,t)$ are the base displacement and the transverse displacement of the beam relative toits base.Based on the proportional damping assumption, thevibration response relative to the base of the bimorphcan be represented as an absolutely anduniformly convergent series of the eigenfunctions as

$$w_{rel}(x,t)=\sum_{r=1}^{\infty}\varphi_r(x)\eta_r(t) \tag{3}$$

Where$\varphi_r(x)$ is the mass normalized eigenfunction of the r-thvibration mode,$\eta_r(t)$ is the modal mechanicalresponse expressions. The eigenfunction representing r-th mode shape corresponding to the undamped free vibration can be described as

$$\varphi_r(x)=A_r[\cos\beta_r x-\cosh\beta_r x+\varsigma_r(\sin\beta_r x-\sinh\beta_r x)] \tag{4}$$

Then, $\varsigma_r$is obtained from

$$\varsigma_r = \frac{m(\sin\beta_r L - \sinh\beta_r L) + \beta_r M_t(\cos\beta_r L - \cosh\beta_r L)}{m(\cos\beta_r L + \cosh\beta_r L) - \beta_r M_t(\sin\beta_r L - \sinh\beta_r L)} \tag{5}$$

3

Where A_r is the constant of modal amplitude, M_t is the weight of tip mass and L is the length of the beam. And the undampednatural frequency of the r-th vibration mode can be expressed as

$$\omega_r = \beta_r^{\ 2}\sqrt{\frac{YI}{m}} \tag{6}$$

And the bending stiffness term YIcan be obtained from

$$YI = \frac{b}{12}(Y_s h_s^{\ 3} + 8c_{11}^E h_p^{\ 3} + 12c_{11}^E h_p^{\ 2} h_s + 6c_{11}^E h_p h_s^{\ 2}) \tag{7}$$

Here,h_s, h_p are the thickness of substrate and piezoelectric layer, respectively. And b is the width of the beam.The constitutive equationwhich relates the electrical andmechanical term for energy harvesting system is

$$\begin{cases} S_1^p = s_{11}^E T_1^p + d_{31}E_3 \\ D_3 = d_{31}T_1^p + \varepsilon_{33}E_3 \end{cases} \tag{8}$$

AndD_3,d_{31}, T_1, S_1, s_{11}^E, ε_{33} and E_3 arethe electrical displacement, the piezoelectric strain constant,the longitudinal stress, the elastic complianceat constantelectric field, the longitudinal strain, the permittivity and the component of applied electric field, respectively.

There are two kinds of longitudinal stress which can be written as the following expressions from equation (8).

$$T_1^s = Y_s S_1^s \tag{9}$$

$$T_1^p = Y_p(S_1^p - d_{31}E_3) \tag{10}$$

Here, Y is the Young's modulus and Y_p=1/ s_{11}^E .1 and 3 directions are coincident with x and y directions, respectively (where 1 is the axial strain direction and 3 is the polarizationdirection).

The distribution of longitudinal bending moment can be obtained as

$$M(x,t) = \int_{a-h_s}^{a} T_1^s bzdz + \int_a^{a+h_p} T_1^p bzdz \tag{11}$$

And a is the distance between neutral layer and the position of the bottom of the piezoelectric layer. The distribution of longitudinal strain can be expressed as

$$S_1(x,y,t) = -y\frac{\partial^2 w_{rel}(x,t)}{\partial x^2} \tag{12}$$

The electric charge q (t)generated in the piezoelectric layer and collected bythe electrodes can be obtained by integrating the electric displacementover the electrode area as [16]

$$q(t) = \int_A \mathbf{D}\cdot\mathbf{n}dA \tag{13}$$

Where **D** is the vector of electric displacements and **n** is the unit normal vector.Then, the outputvoltage generated by the piezoelectric materials can be given by

$$v(t) = R_{load}i(t) = R_{load}\frac{dq(t)}{dt} \tag{14}$$

Where $i(t)$ is current of theelectric circuit. Furthermore, the output voltage density can be obtained as

$$\rho_{v(t)} = \frac{v(t)}{V_s + V_p + V_t} \tag{15}$$

Here, V_s, V_p andV_t represent the volume of substrate materials, piezoelectric materials and tip mass, respectively.

PZT-5H and copper are used to produce the beam and the physical attributes of original piezoelectric cantilever are shown in table 1.

Table 1. Physical attribute parameters of piezoelectric cantilever.

Related parameters	Piezoelectric ceramic	Copper substrate
$Y(GPa)$	60.61	110
$d_{31}\,(C{\cdot}N^{-1})$	-3.2×10^{-10}	—
$\varepsilon_{33}\,(F{\cdot}m^{-1})$	3.02×10^{-8}	—
L (mm)	70.0	70.0
b (mm)	15.0	15.0
$h_s(h_p)$ (mm)	2.0	1.0

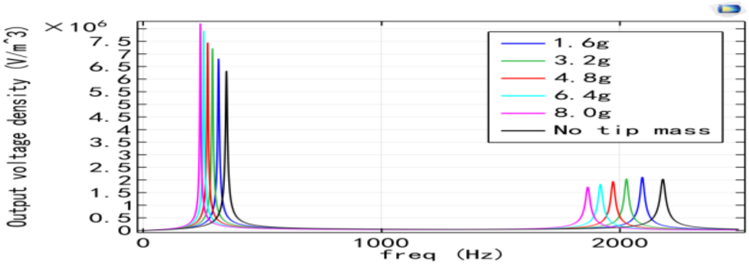
III. Voltage Density Response Of External Factors

The effect of external factors, such as tip mass, acceleration of base excitation and load resistance are of highly significance to maximize vlotage output. Accordingly, the analysis of influences of external factors on output performance is highly significant for achieving more desirable electricity energy. Additionally, the output performance of energy transducers is evaluated with voltage density (including the volume of tip mass), as mentioned in equation (15).

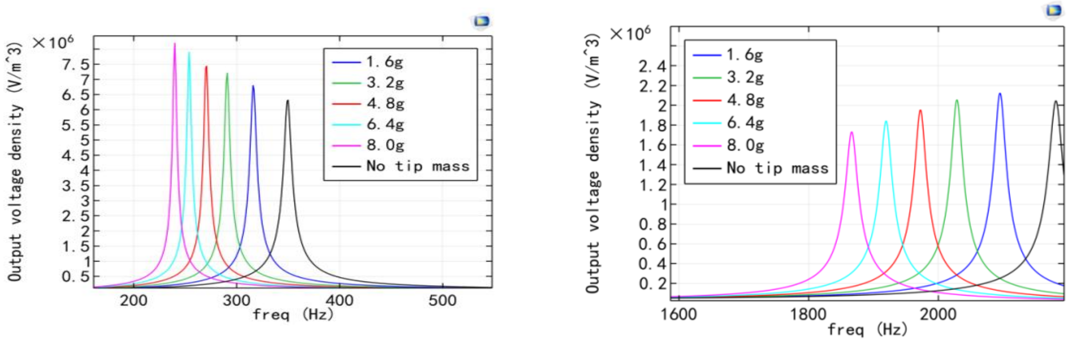
3.1 Frequency response of transducers with or without a tip mass

Figure 2 exhibits the effect of tip mass on harvesting voltage density for piezoelectric cantilevers. The simulation process is carried out using fivedifferent tip mass: 0, 1.6g, 3.2g, 4.8g, 6.4g and 8.0g. it can be found that, on one hand, the voltage density generated at the first mode increases along with the growing of the weight of the tip mass. Meanwhile, the first naturalfrequency decreases as the weight of the proof mass increased.On the other hand, it is contrary to the first mode that the voltage density produced at the second mode increases with the decrease of the weight of the tip massexcept energy transducers without tip mass which exhibit a relatively high output density performance. Moreover, it can be found that the voltage density of energy transducers with 8.0g tip mass increase by 30.2% compared with no tip mass ones at the first natural frequency.On the contrary, the output density of energy transducers with no tip mass increase by 17.2% compared with 8.0g tip mass ones at the second natural frequency.

Hence, the tip mass has the ability to tune the natural frequencyof the beam. So, in order to obtain better voltage density, increasing the weight of tip mass properly when excitation frequency is around the first natural frequency or reducing the weight when it comes to the second natural frequency.



(a)



(b) (c)

Figure 2.(a) Frequency response of output voltage density with different tip mass, (b)close up for the first mode, (c)close up for the second mode.

3.2 Acceleration response

The acceleration of external base excitation is also of great influence on piezoelectric energy transducers. The process is carried out using 8.0g tip mass at the first natural frenquency (240 Hz). Here, as demonstrated in figure 3, the acceleration is set from 0.25*g* to 2.0*g* , where *g* is the gravitational acceleration (g=9.81 m/s^2). It can be seen clearly that the output voltage density increases in proportion with the growing of the acceleration of external base excitation. Furthermore, the proportion between the output voltage density and the acceleration is found to be $0.81\times10^7 V/(m^3\cdot g)$, which shows that higher excitationacceleration can lead to more desirable output voltage density.

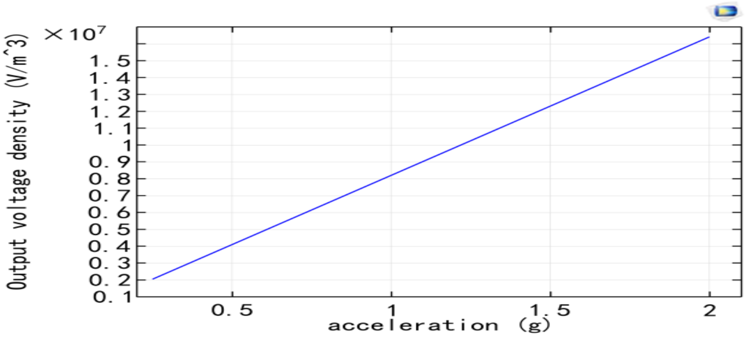


Figure 3. Acceleration response of output voltage density using 8.0g tip masss at 240Hz.

3.3 Load resistance response

The electric circuit of energy harvesting system consists of structure of piezoelectric beam and external load resistance. Figure 4presents the relationships among the load resistance (from 10Ω to 1000kΩ), output voltage density and normalized output power density (output power density/8) using 8.0g tip masss at its first natural frenquency 240Hz. For the purpose of revealing the change law of output voltage/power density along with different load resistance, logarithmic *x*coordinate is used to exhibit clearly. It is evident that output voltage density increases rapidly when load resistance is relatively small, it increases slowlywhen load resistance is relatively large. When load resistance is more than about 40kΩ, the output voltage density curve is almost a straight line. On the contrary, there exists an peak of output power density with various external load resistance which found to be 3.16kΩ.

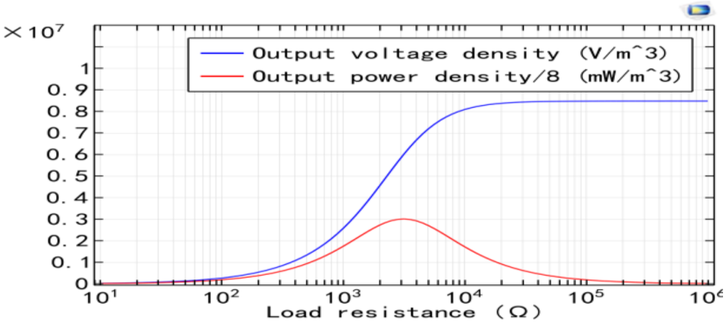
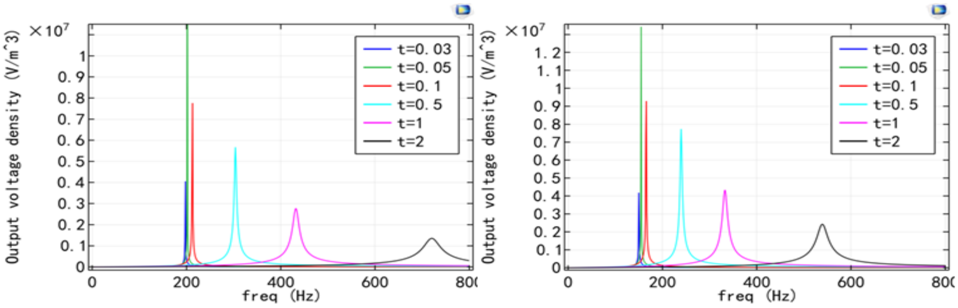


Figure 4. Load resistance response of output voltage/power densityusing 8.0g tip masss at 240Hz.

IV. Voltage Density Response Of Internal Factors

Besides external factors, internal factors such as thickness ratio of piezoelectric layer and substrate layer ($\alpha = h_s/h_p$), material properties of substrate have distinct influences on output voltage density. Here, as shown in figure 5, two kinds of most commonly used metal materials, aluminum and copper are taken into consideration of substrate material. It can be found that there always exists an optimal thickness ratio corresponding to the most desirable output voltage density either aluminum or copper substrate involved in energy harvesting systems.In addition, the first natural frequency increases along with the growing of thickness ratio and the reason is that more metal components (bigger thickness ratio) increase the equivalentstiffness of the systems. In terms of output voltage density at the first natural frequency of various thickness ratio, the performance increases firstly then decreases with growing thickness ratio.Moreover, From a global view, cantilevered piezoelectric energy transducers using copper substrate generated more output voltage density than the ones with aluminum substrate at the first natural frequency.



(a) (b)

Figure 5. Comparison of output voltage density with different thickness ratio,
(a) aluminum substrate, (b)copper substrate.

V. Conclusions

The models of unimorph cantilevers are established to evaluate the harvesting performance of piezoelectric transducers under finite volume,and then the analyses of influences of external and internal factors on harvesting energy density is proposed. The results show that the more heavy the tip mass is, the better output performance it would be at the first natural frequency, which is contrary to output capacity at the second natural frequency. Moreover, the harvesting voltage density is proportional to the acceleration of the base excitation, and the proportion between the output voltage density and the acceleration is found to be $0.81 \times 10^7 V/(m^3 \cdot g)$, which shows that higher excitation acceleration can lead to more desirable output voltage density. In terms of external load resistance of the electric circuit, it is found that output voltage density increases along with the growth of load resistance and there exists an optimal load resistance which leads to the most desirable output power density.

Besides external factors, internal factors such as thickness ratio and material properties of substrate can be of highlyimportance to output density performance. The findings reveal that an optimum thickness ratio corresponding to the most desirable output voltage density would emerge either aluminum or copper substrate involved in energy harvesting systems and further studies apparently show that copper substrate generates higher performance in comparison with aluminum substrate.

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